String and String-Inspired Phenomenology ¹

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ABSTRACT

In these lectures I review the progress made over the last few years in the subject of string and string-inspired phenomenology. I take a practical approach, thereby concentrating more on explicit examples rather than on formal developments. Topics covered include: introduction to string theory, the free-fermionic formulation and its general features, generic conformal field theory properties, $SU(5)\times U(1)$ GUT and string model-building, supersymmetry breaking, the bottom-up approach to string-inspired models, radiative electroweak symmetry breaking, the determination of the allowed parameter space of supergravity models and the experimental constraints on this class of models, and prospects for direct and indirect tests of string-inspired models.

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Contents

1	Introduction						
	1.1 Why strings?	1					
	1.2 What are strings?	2					
2	String basics						
3	3 Free-fermionic formulation						
4	General results in free-fermionic models	6					
	4.1 Gravity is always present	6					
	4.2 Spacetime supersymmetry	7					
	4.3 Gauge groups	8					
	4.4 Superpotential	8					
5	Generic conformal field theory properties	g					
6	SU(5)xU(1) GUT model-building	11					
7	$\mathrm{SU}(5)\mathrm{xU}(1)$ string model-building	13					
8	Supersymmetry breaking	18					
9	The bottom-up approach	19					
	9.1 Unification of gauge couplings	20					
	9.2 Soft supersymmetry breaking	20					
	9.3 Parameter count at low energies	21					
	9.4 Determination of the theoretically allowed parameter space	22					
10	Radiative Electroweak Breaking	23					
	10.1 Tree-level minimization	23					
	10.2 One-loop minimization	23					
	10.3 Radiative Symmetry Breaking	24					
11	The allowed parameter space	2 5					
12	Experimental constraints	25					
	12.1 $b \rightarrow s\gamma$	26					
	12.2 $(g-2)_{\mu}$	26					
	$12.3 \epsilon_1 \epsilon_2$	27					

12.4 Neutrino telescopes	27
13 Prospects for direct detection	28
13.1 Tevatron	28
13.2 LEPII	28
13.2.1 Lightest Higgs boson	28
13.2.2 Charginos	29
13.2.3 Sleptons	29
13.3 DiTevatron	30
14 Conclusions	30

1 Introduction

The quest for a "Theory of Everything" has captured the imagination of many physicists over the years. Unfortunately, such an enterprise is by definition very ambitious and often driven by grand principles rather than by compelling experimental information. In fact, progress can be made at the fastest pace when both bright theoretical insights and clear experimental data are available simultaneously, such as in the development of Quantum Mechanics in the early part of this century, and more recently in the elucidation of the theory of strong and electroweak interactions. The larger theory that is to contain the Standard Model is in the process of being elaborated at this time, but experimental data are not helping in the traditional way since they agree with the Standard Model predictions very well. Nonetheless, physicists believe that some larger theory must exist.

The search for clues as to the nature of this all-encompassing theory has followed a path towards larger and larger energies, *i.e.*, grand unification, supersymmetry, supergravity, and superstrings. This line of thought indicates that the explanation for all observable phenomena is to be found in the theory of superstrings. Superstrings have so far one undisputable success: they provide the only known consistent theory of quantum gravity. However, for physics at low energies, the more interesting aspect of string theory is its possible explanation of the Standard Model. This aspect of string theory has developed over the last several years and is the main subject of these lectures.

1.1 Why strings?

Since string theory is so complicated and as yet still only mildly understood, the motivations for investing a great deal of time exploring it have to be spelled out in as clear a way as possible.² Below I give a logical path towards string theory starting from the Standard Model. Pluses indicated successes, whereas minuses indicate problems which force us to keep going down the list.

• Standard Model:

- + Experimentally very successful (Higgs boson?).
- Many unexplained features: $N_g = 3$, fermion masses, quark mixings, ...

• Unified theories:

- + Answer some of the Standard Model puzzles (charge quantization, fermion mass relations, neutrino masses).
- Gauge hierarchy problem: why is $M_W \ll M_U$?

²I should note that the more theoretically inclined students seem to find enough motivation to study string theory in its intriguing and partly unknown mathematical structure.

- Supersymmetry:
 - + Solves gauge hierarchy problem.
 - Supersymmetry breaking: why should sparticles be ≤ TeV so that the gauge hierarchy problem is not reintroduced?
- Supergravity:
 - + Supersymmetry breaking becomes calculable in terms of few inputs.
 - What determines the inputs to the supergravity model? What about quantum gravity?
- Superstrings:
 - + Finite theory of quantum gravity, *everything* predicted in a given vacuum ("model").
 - ? Many possible vacua. What selects the "correct" vacuum?

1.2 What are strings?

As a way of introduction, let us list several string characteristics:

- \bullet One-dimensional extended objects: $\sim 10^{-33}$ cm in length.
- Particles are identified with various string modes:
 - massless modes which should contain the Standard Model particles, and
 - infinite tower of massive modes ($\sim M_{Pl}$ and higher).
- Rather stringent consistency conditions (conformal invariance, modular invariance) restrict the type of allowed string theories: bosonic, heterotic, type II.
- Heterotic string: closed string theory with two different choices for the left- and right-moving string modes (supersymmetric and non-supersymmetric). Most promising phenomenologically.
- Number of degrees of freedom on the string is constrained. In the simplest case consistency requires:
 - non-supersymmetric string lives in 26 dimensions, and
 - superstring lives in 10 dimensions.
- For the heterotic string in 10 dimensions, only two gauge groups are allowed: $E_8 \times E_8$ and SO(32).
- Must "compactify" extra six dimensions (Calabi-Yau manifolds, orbifolds), or can construct theories directly in four dimensions.

- There are many (∞ ?) four-dimensional models (compactified or not). All these are allowed vacua of the theory.
- Precise model-building rules give gauge group, spectrum, and interactions for any given vacuum or "model". This subject is called "string phenomenology".

2 String basics

- The one-dimensional string sweeps out a two-dimensional world-sheet embedded in *D*-dimensional spacetime.³
- The two-dimensional action describes the dynamical evolution in terms of bosonic (X^{μ}) and fermionic (ψ^{μ}) fields on the world-sheet.
- The classical solutions to the string equations of motion can be expanded in terms of "left-moving" and "right-moving" modes. The two sectors are basically decoupled (except that they must contribute equally to the mass of the string) and can be chosen to be different theories (*i.e.*, with or without world-sheet supersymmetry):
 - Bosonic string: non-supersymmetric⊗non-supersymmetric,
 - Type II string: supersymmetric⊗supersymmetric,
 - Heterotic string: supersymmetric⊗non-supersymmetric.
- The two-dimensional free-field action is conformal invariant at classical level. However, conformal anomalies appear after quantization, with each world-sheet field contributing a specific amount to the anomaly:
 - each boson: c=1
 - each fermion: $c = \frac{1}{2}$
 - Fadeev-Popov ghosts: c = -26
 - Fadeev-Popov superghosts: c = 11

The Fadeev-Popov ghosts appear in the quantization of the non-supersymmetric string, whereas the superghosts appear additionally in quantizing the supersymmetric string. In a consistent string theory the total contribution to the conformal anomaly (c_{tot}) must vanish.

• Bosonic string: $c_{\rm tot} = 1 \cdot D_c^b + (-26) = 0 \Rightarrow D_c^b = 26 \ X^{\mu}$ fields required, *i.e.*, the bosonic string lives in 26 dimensions.

³For a textbook introduction to string theory see Ref. [1].

- Fermionic string: $c_{\text{tot}} = 1 \cdot D_c^f + \frac{1}{2} \cdot D_c^f + (-26) + 11 = 0 \Rightarrow D_c^f = 10 \ (X^{\mu}, \psi^{\mu})$ pairs required, *i.e.*, the heterotic and type II strings live in 10 dimensions.
- If some of the degrees of freedom on the world-sheet are interpreted as "internal" (not spacetime), the actual required dimension is lower than the "critical" dimension calculated above, i.e., $D < D_c$.
- Figure 1 shows how external string states are conformally mapped onto the world-sheet and are represented by vertex operators. These operators encode all of the quatum numbers of the string state.
- String perturbation theory is an expansion in the topology of the two-dimensional world-sheet (see Fig. 2).
- Topologically distinct surfaces have different number of handles ("genus"). Thus, at each order in perturbation theory there is only one string diagram. This in contrast with the large number of diagrams present at high orders in regular quantum field theory.⁴
- String scattering amplitudes are defined as a path integral over the two-dimensional quantum field theory on the world-sheet, with insertions of suitable vertex operators representing the particles being scattered. This recipe takes into account automatically the infinite number of massive modes which could be exchanged in the scattering process.
- In modern language this corresponds to calculating correlation functions of vertex operators in the two-dimensional conformal field theory.

3 Free-fermionic formulation

- The idea is to formulate string theory directly in four dimensions. This requires additional degrees of freedom on the world-sheet to cancel the conformal anomaly. In the free-fermionic formulation [3] these are chosen to be free world-sheet fermions (with $c = \frac{1}{2}$ each). For the heterotic string:
 - left-movers (supersymmetric): $1 \cdot D + \frac{1}{2} \cdot D + \frac{1}{2} \cdot n_L + (-26) + 11 = 0 \Rightarrow n_L = 18$ left-moving real fermions.
 - right-movers (non-supersymmetric): $1 \cdot D + \frac{1}{2} \cdot n_L + (-26) = 0 \Rightarrow n_R = 44$ right-moving real fermions.
- The two-dimensional world-sheet fields are:
 - left-movers: X^{μ} , ψ^{μ} , $(\chi^i, y^i, w^i)^{i=1\rightarrow 6}$

⁴This stringy property has been exploited to compute complicated QCD processes in the Standard Model in a much simplified way, by viewing QCD as a low energy limit of string theory [2].

- right-movers: \bar{X}^{μ} , $\bar{\phi}^{1\rightarrow 44}$
- States in the Hilbert space are constructed by acting on the vacuum with creation operators associated with the ordinary $(X^{\mu}, \psi^{\mu}, \bar{X}^{\mu})$ and free-fermionic world-sheet fields, e.g., $\psi^{\mu}_{\frac{1}{2}}\bar{X}^{\nu}_{1}|0\rangle$.
- The frequencies of the creation operators determine the mass of the string state, and their nature depends on the boundary conditions of the free fermions as they are parallel transported around loops on the (one-loop) world-sheet (torus). In the simplest case these boundary conditions can be periodic or antiperiodic. Periodic boundary conditions imply integer frequencies ("Ramond sector"); whereas antiperiodic boundary conditions imply half-integer frequencies ("Neveu-Schwarz sector").
- The partition function of the world-sheet fermions depends on these boundary conditions (these are called "spin structures"). Constraints on the spin structures follow by demanding world-sheet modular invariance, i.e., physics should be independent of the two-dimensional surfaces being cut and reconnected. Spin structures are specified for each world-sheet fermion and are all collected in "vectors" with 2+18 left-moving and 44 right-moving entries. In these vectors 1 (0) entries represent periodic (antiperiodic) boundary conditions.
- Further constraints imply that certain states must be dropped from the spectrum. These states are eliminated by a set of generalized "GSO projections".
- A consistent vacuum or "model" is specified by:
 - A basis for the spin-structure vectors $\{b_1, b_2, \dots, b_n\}$.
 - An $n \times n$ matrix of GSO projections $C\begin{pmatrix} b_i \\ b_j \end{pmatrix}$.
 - One must also verify that several consistency conditions are satisfied for both allowed basis vectors and allowed GSO projection matrices.
 - The basis vectors span a space of "sectors" of the Hilbert space, which contain the allowed physical states. For a given state, the GSO projection may or may not project it out.
 - For a state in a given sector α (a linear combination of the basis vectors) its mass is given by $M^2 = -1/2 + (1/8)\alpha_L^2 + N_L = -1 + (1/8)\alpha_R^2 + N_R$, where $N_L(N_R)$ is the sum of the frequencies of the left-(right-)moving oscillators which create the state and $\alpha_{L(R)}^2$ is the length-squared of the left-(right-) moving part of α .
- These model-building rules lead to numerous possible models:
 - the b_i have 22+44 entries,

- the $n \times n$ GSO-projection matrix has n(n-1)/2 independent elements,
- a typical model consists of n = 8 vectors, i.e., $2^{(8.7)/2} = 268$ million choices.
- There is a large amount of redundancy in the models so constructed. Morever, many possibilities are ruled out on phenomenological grounds, e.g., no spacetime supersymmetry, more or less than three generations of quarks and leptons, no gauge group than can be broken down to the Standard Model, etc.

4 General results in free-fermionic models

The free-fermionic formulation described above allows one to construct in a systematic way large numbers of string models. The phenomenological properties of these models can vary a lot from model to model, although there are many models with nearly identical properties. We now list a few properties which are generic in large classes of models of this kind.

4.1 Gravity is always present

- The simplest basis contains only one vector: $\{b_1 = 1\}$, *i.e.*, all fermions are periodic.
- There are two sectors: b_1 which only contains massive states, and $2b_1 = 0$ which is the Neveu-Schwarz sector.
- Massless spectrum $(M^2 = 0)$:

$$\begin{array}{ll} \psi^{\mu}_{\frac{1}{2}}\bar{X}^{\nu}_{1}|0\rangle_{0} & \begin{cases} \text{graviton} \\ \text{dilaton} \\ \text{antisymmetric tensor} \end{cases} \\ \psi^{\mu}_{\frac{1}{2}}\bar{\phi}^{a}_{\frac{1}{2}}\bar{\phi}^{b}_{\frac{1}{2}}|0\rangle_{0} & \text{gauge bosons of } SO(44) \\ (\chi^{i}_{\frac{1}{2}},y^{i}_{\frac{1}{2}},w^{i}_{\frac{1}{2}})\bar{X}^{\mu}_{1}|0\rangle_{0} & \text{gauge bosons of } SU(2)^{6} \\ (\chi^{i}_{\frac{1}{2}},y^{i}_{\frac{1}{2}},w^{i}_{\frac{1}{2}})\bar{\phi}^{a}_{\frac{1}{2}}\bar{\phi}^{b}_{\frac{1}{2}}|0\rangle_{0} & \text{scalars in adjoint of } SU(2)^{6} \times SO(44) \end{cases}$$

These states are all allowed by the GSO projections. Therefore, the graviton is always present in this type of models. Furthermore, gauge interactions are also generically present.

- There is also a tachyon: $\bar{\phi}_{\frac{1}{2}}^{a}|0\rangle_{0}$ (with $M^{2}<0$), since this is not a supersymmetric model (there are no massless fermions).
- \bullet The vacuum state $|0\rangle_0$ is the non-degenerate, spin-0, Neveu-Schwarz vacuum.

4.2 Spacetime supersymmetry

• The next-to-simplest model has basis $\{b_1, S\}$, with

$$S = \psi^{\mu} \quad \chi^{1} \quad y^{1} \quad w^{1} \quad \chi^{2} \quad y^{2} \quad w^{2} \quad \cdots \quad \chi^{6} \quad y^{6} \quad w^{6}$$

$$1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad \cdots \quad 1 \quad 0 \quad 0$$

and 0 for all the right-movers.

- There are four sectors: b_1 , $b_1 + S$, 0, S.
- S is the supersymmetry "generator": the states in $b_1 + S$ are the superpartners of those in b_1 (all are massive), whereas the states in S are the superpartners of the Neveu-Schwarz states in 0.
- Massless spectrum:

The model contains the complete N=4 supermultiplet with four gravitinos, thus the model has N=4 supersymmetry. Also, the gauge particles form a complete N=4 supermultiplet, as they should. (The multiplicity of states in these multiplets is indicated in the # column.)

- The $|0\rangle_S$ vacuum state is the degenerate, spin-1/2, Ramond vacuum. This state is built from the eight periodic fermions in S (*i.e.*, ψ^{μ} , $\chi^{1\to 6}$) which transform as the irreducible spinor representation of SO(8) (*i.e.*, 4 Weyl fermions).
- The vector S brings in new GSO projections which *eliminate* the tachyon from the spectrum, as it should be in a supersymmetric model.
- With addition of further vectors to the basis, it is possible to reduce the number of spacetime supersymmetries from 4 to 2 and then to 1. This is a more complicated, although straightforward exercise.

4.3 Gauge groups

- The simply-laced Lie groups SO(2n), SU(n), E_6 , E_7 , E_8 are readily obtainable. Others can also be obtained but in a less straightforward manner. Usually one has to collect gauge boson states from several sectors to deduce the gauge group.
- A typical example is $SU(5) \times SO(10) \times SU(4) \times U(1)^n$.
- In general, in first approximation all gauge couplings are unified at the string scale $M_{\text{string}} \approx 5 \times g \times 10^{17} \,\text{GeV}$, where g is the unified coupling [4]. String threshold corrections can split the various gauge couplings by a model-dependent amount [5].
- A very special property of string theory in general is that the gauge coupling is determined dynamically as the expectation value of the dilaton field $S: g^2 \propto 1/\langle S \rangle$.
- Models built in any formulation consist of sets of matter representations which
 are automatically anomaly free. In practice this is a good check of the derivation
 of complicated models.

4.4 Superpotential

- In the free-fermionic formulation the superpotential is calculable to any order in the string fields by using the techniques of conformal field theory [6]. Generally one obtains
 - Cubic terms: $\lambda \phi_1 \phi_2 \phi_3$, with $\lambda = c_3 g$ and $c_3 = \{\frac{1}{2}, \frac{1}{\sqrt{2}}, 1\}$.
 - Non-renormalizable terms: $\lambda \phi_1 \phi_2 \dots \phi_N \frac{1}{M^{N-3}}$, with $\lambda = c_N g$ and $c_N \sim 1$ a calculable coefficient, and $M \approx 10^{18}$ GeV.
- Non-trivial calculational techniques are required for the non-renormalizable terms because of the coupling between left- and right-moving degrees of freedom on the world-sheet. This property is related to the asymmetric orbifold character of this class of models.
- Non-renormalizable terms provide a natural hierarchical fermion mass scenario [7]:

$$\lambda Q_3 t^c H, \qquad \lambda_t \sim g;$$

$$\lambda Q_2 c^c H \frac{\langle \phi \rangle}{M}, \qquad \lambda_c \sim g \frac{\langle \phi \rangle}{M};$$

$$\lambda Q_1 u^c H \frac{\langle \phi \rangle^2}{M^2}, \quad \lambda_u \sim g \frac{\langle \phi \rangle^2}{M^2}$$

If $\langle \phi \rangle / M < 1$, as is motivated by the cancellation of an anomalous $U_A(1)$ symmetry always present in these models, then a hierarchy of Yukawa couplings can be obtained.

5 Generic conformal field theory properties

- All spacetime symmetries in string theory have their origin in world-sheet symmetries.
- Kac-Moody algebras: [8]
 - The affine Kac-Moody algebra \widehat{G} underlies the spacetime gauge symmetry G. This algebra is represented by currents made of world-sheet fermions, and can be realized at different positive integer levels k. All states in the theory fall into representations of this algebra.
 - For a fixed level k, only certain representations to the gauge group (G) are unitary and thus allowed. These representations must satisfy

$$\sum_{i=1}^{\operatorname{rank} G} n_i m_i \le k \,,$$

where the n_i are the Dynkin labels of the highest weight representation, and the m_i are sets of numbers that depend on the gauge group,

	m_i		
SO(2n)	$(1, 2, 2, \dots, 2, 1, 1)$		
SU(n)	$(1,1,1,\ldots,1)$		
E_6	(1, 2, 3, 2, 1, 2)		
E_7	(2, 3, 4, 3, 2, 1, 2)		
E_8	(2,3,4,5,6,4,2,3)		

- Kac-Moody algebras (i.e., the degrees of freedom that they represent) contribute to the central charge c of the theory

$$c = \frac{k \dim G}{k + \tilde{h}} \;,$$

where $\tilde{h} = \frac{1}{2}C_A$ is the dual Coxeter number, and C_A the quadratic Casimir of the adjoint representation.

- For the non-supersymmetric right-movers, the conformal anomaly cancellation equation is $4 + c_{\text{matter}} + (-26) = 0 \Rightarrow c_{\text{matter}} = 22$. For a product group $G = \prod_i G_i$, the constraint is stronger: $\sum_i c_i = 22$.
- Since the right-movers are responsible for representing the gauge group of the string model, the corresponding Kac-Moody algebra should have a central charge with $c \leq 22$. This entails an upper bound on the allowed level

$$k \le \left[22\tilde{h}/(\dim G - 22)\right].$$

For various groups of interest we get

$$\begin{array}{ll} SU(5) & k \le 55 \\ SO(10) & k \le 7 \\ E_6 & k \le 4 \\ E_7 & k \le 3 \\ E_8 & k \le 2 \end{array}$$

• An allowed representation (r) at level k is massless if $h_r \leq 1$, where

$$h_r = \frac{C_r}{2k + C_A}$$

is its conformal dimension (C_r is the quadratic Casimir of the representation⁵).

- Combining the constraints from unitarity and masslessness, significant restrictions follow on the allowed gauge group representations [10].
- Unitary massless representations at level 1
 - SO(2n): singlet, vector, and spinor. Spinor massless for $n \leq 8$ only.
 - -SU(n): totally antisymmetric representations (see table).

n	Representation
2	1,2
3	$1,3,\overline{3}$
4	$1,4,\overline{4},6$
5	$1,5,\overline{5},10,\overline{10}$
6	$1,6,\overline{6},15,\overline{15},20$
7	$1,7,\overline{7},21,21,35,\overline{35}$
8	$1,8,\overline{8},28,\overline{28},56,\overline{56},70$
9	$1,9,\overline{9},36,\overline{36},84,\overline{84}$
10-23	$1,$ n $,$ $\overline{\mathrm{n}},$ n $(\mathrm{n-1})/2,$ $\overline{\mathrm{n}(\mathrm{n-1})/2}$

$$-E_6$$
: 1, 27, $\overline{27}$

 $-E_7$: **1**, **56**

- E_8 : **1**
- Note that there are no adjoint representations allowed at level 1.
- What levels would be required for traditional GUT model building?

⁵The group theoretical constants mentioned here $(n_i, C_A, C_r, \text{ etc.})$ have been tabulated [9].

- SU(5): for doublet-triplet splitting through the missing partner mechanism one requires $50,\overline{50},75$ (the 75 breaks the gauge symmetry). These are unitary at level $k \geq 2$, but massless for $k \geq 4$. The unitary and massless representations at k = 4 are: $1,5,\overline{5},10,\overline{10},15,24,40,\overline{40},45,\overline{45},50,\overline{50},75$. Therefore, very likely lots of exotics in the models.
- SO(10): requires $10,16,45,54,126,\overline{126}$ [11]. All these are unitary and massless for $k \geq 5$, but so are the $144,\overline{144},210$.
- String model-building using level-one Kac-Moody algebras includes almost every string model ever built. Model-building using higher-level Kac-Moody algebras has been very limited because of the technical difficulties involved [12]. The proliferation of exotic representations is also a potential problem.
- At level-one, the $SU(5) \times U(1)$ gauge group becomes singled out because the gauge symmetry is broken by $\mathbf{10}, \overline{\mathbf{10}}$ representations [13], which are allowed and occur in all known models of this kind. There exist also string constructions in the free-fermionic formulation where the Pati-Salam gauge group $SU(4) \times SU(2) \times SU(2)$ is obtained [14], or even the Standard Model gauge group itself [15, 16].

$6 \quad SU(5)xU(1) GUT model-building$

From the previous section we conclude that $SU(5) \times U(1)$ is an interesting candidate for a string-derived gauge group. This gauge group is also quite attractive from the traditional GUT (non-string) model-building perspective, as we know recollect.

• The Higgs and matter fields are in the following representations:

- **10**:
$$H = \{Q_H, d_H^c, \nu_H^c\}, \overline{\mathbf{10}}: \bar{H} = \{Q_{\bar{H}}, d_{\bar{H}}^c, \nu_{\bar{H}}^c\},$$

- **5**: $h = \{H_2, H_3\}, \overline{\mathbf{5}}: \bar{h} = \{\bar{H}_2, \bar{H}_3\}.$
- **10**: $F_i = \{Q, d^c, \nu^c\}_i, \overline{\mathbf{5}}: \bar{f}_i = \{L, u^c\}_i, \mathbf{1}: l_i = e_i^c.$

• The GUT superpotential is assumed to be:

$$W_G = H \cdot H \cdot h + \bar{H} \cdot \bar{H} \cdot \bar{h} + F \cdot \bar{H} \cdot \phi + \mu h \bar{h},$$

where the vacuum expectation values of the neutral components of the H and \bar{H} fields $(\langle \nu_H^c \rangle = \langle \nu_{\bar{H}}^c \rangle = M_U)$ break $SU(5) \times U(1)$ down to $SU(3) \times SU(2) \times U(1)$.

• Doublet-triplet splitting: the Higgs pentaplets (h, \bar{h}) have two components with very different purposes

$$h = \begin{pmatrix} H_2 \\ H_3 \end{pmatrix}$$
 electroweak symmetry breaking proton decay

The mass splitting of the doublets and triplets is accomplished by the following superpotential interactions

$$\begin{split} H \cdot H \cdot h &\to d_H^c \left\langle \nu_H^c \right\rangle H_3 \\ \bar{H} \cdot \bar{H} \cdot \bar{h} &\to \bar{d}_H^c \left\langle \bar{\nu}_H^c \right\rangle \bar{H}_3 \end{split}$$

whereby the triplets get heavy, whereas the doublets remain light. This phenomenon is a manifestation of the "missing partner mechanism" [17]. (A similar mechanism in SU(5) requires the introduction of large representations for this sole purpose.)

• The Yukawa part of the superpotential is given by

$$\lambda_d F \cdot F \cdot h + \lambda_u F \cdot \bar{f} \cdot \bar{h} + \lambda_e \bar{f} \cdot l^c \cdot h$$

and generates the fermion masses. Note that unlike SU(5), there is no $m_b = m_\tau$ relation in $SU(5) \times U(1)$.

• Neutrino masses follow from a generalized see-saw mechanism:

$$\begin{cases}
F \cdot \bar{f} \cdot h \to m_u \nu \nu^c \\
F \cdot \bar{H} \cdot \phi \to \langle \nu_{\bar{H}}^c \rangle \nu^c \phi
\end{cases}
M_{\nu} = \begin{array}{c}
\nu \\ \nu \\
\nu \\
\phi
\end{array}
\begin{pmatrix}
0 \\ m_u \\ 0 \\ M_U
\end{pmatrix}$$

This mechanism gives $m_{\nu_{e,\mu,\tau}} \sim m_{u,c,t}^2/M_U$, which have been shown to be consistent with the MSW mechanism, ν_{τ} dark matter, and (ν^c) baryogenesis [18].

- Proton decay through dimension-six operators is mediated by heavy gauge bosons, and is highly suppressed since $M_U \sim 10^{18} \, \mathrm{GeV}$.
- Proton decay mediated by dimension-five operators (see Fig. 3)

$$\lambda_d F \cdot F \cdot h \supset QQH_3 \qquad \lambda_u F \cdot \bar{f} \cdot \bar{h} \supset QL\bar{H}_3$$

is very suppressed since no H_3 , \bar{H}_3 mixing exists, even though H_3 , \bar{H}_3 are heavy via the doublet-triplet splitting mechanism [19].

SU(5)xU(1) string model-building 7

We now turn to the actual string models containing the gauge group $SU(5) \times U(1)$ which have been built within the free-fermionic formulation.

- There are two variants of the model:
 - the "revamped" model, Antoniadis-Ellis-Hagelin-Nanopoulos (1989) [20], and
 - the "search" model, Lopez-Nanopoulos-Yuan (1992) [21].

These models have the following gauge group

$$\underbrace{SU(5) \times U(1)}_{observable} \times \underbrace{U(1)^n}_{mixed} \times \underbrace{SO(10) \times SU(4)}_{hidden}$$

with n = 4(5) for the "revamped" ("search") model.

• The observable sector spectrum is schematically given by

"Revamped"

(Both models also contain 1 representations.)

- Representations labelled "EV" contain new, vector-like, heavy particles, beyond those in the minimal supersymmetric standard model (MSSM).
- Motivation for the "search" model:
 - the "revamped" model unifies at best at $M_U \sim 10^{16}\,\mathrm{GeV}$
 - the "search" model has new Q, \bar{Q} and D^c, \bar{D}^c representations to push M_U up to the string scale:

if
$$M_U \sim 10^{18} \, \text{GeV}$$
, then $\begin{cases} M_{Q,\bar{Q}} \sim 10^{12} \, \text{GeV} \\ M_{D^c,\bar{D}^c} \sim 10^6 \, \text{GeV} \end{cases}$

- In general, to "work out" a model one needs to study:
 - the Higgs doublet mass matrix,
 - the Higgs triplet mass matrix,
 - the F- and D-flatness constraints in the presence of the anomalous $U_A(1)$,
 - the dimension-five proton decay operators which may be reintroduced in the string models.
- Things are not simple because models contain many singlet fields, which can (and some must) get vacuum expectation values (vevs). Moreover, many entries in the Higgs mass matrices depend on these unknown vevs. Nonetheless, it is possible to have all pieces of the model "work out" for some choices of the vevs. The resulting model is a deformation of the original free-fermionic model.
- Let us now describe one complete model (the "search" model) starting from the inputs (basis vectors and matrix of GSO projections) and giving the results for the massless spectrum, the superpotential, and the Higgs mass matrices.
- Basis vectors: the first entry corresponds to the complexified ψ^{μ} and the next 18 entries to the six left-moving triplets $(\chi^{\ell}, y^{\ell}, \omega^{\ell})$. The first 12 right-moving entries (to the right of the colon) correspond to the real fermions $\bar{y}^{\ell}, \bar{\omega}^{\ell}$, and the last 16 entries correspond to complex fermions. A 1 (0) stands for periodic (antiperiodic) boundary conditions. We also use the symbols $1_8 = 111111111$, $0_8 = 000000000$, $A = \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} 1100$.

• The matrix of GSO projections:

$$k = \begin{pmatrix} 2 & 1 & 2 & 2 & 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 4 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 4 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 \\ 2 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 3 \end{pmatrix}$$

- The massless spectrum (the charges under the five U(1)'s are as indicated)
 - Observable Sector:

$$\begin{array}{lll} F_0\left(-\frac{1}{2},0,0,-\frac{1}{2},0\right) & F_1\left(-\frac{1}{2},0,0,\frac{1}{2},0\right) \\ F_2\left(0,-\frac{1}{2},0,0,0\right) & \bar{f_2}\left(0,-\frac{1}{2},0,0,0\right) & l_2^c\left(0,-\frac{1}{2},0,0,0\right) \\ F_3\left(0,0,\frac{1}{2},0,-\frac{1}{2}\right) & \bar{f_3}\left(0,0,\frac{1}{2},0,\frac{1}{2}\right) & l_3^c\left(0,0,\frac{1}{2},0,\frac{1}{2}\right) \\ F_4\left(-\frac{1}{2},0,0,0,0\right) & \bar{F_4}\left(\frac{1}{2},0,0,0,0\right) & l_5^c\left(0,-\frac{1}{2},0,0,0\right) \\ \bar{F_5}\left(0,\frac{1}{2},0,0,0\right) & \bar{h_1}\left(-1,0,0,0,0\right) & h_2\left(0,1,0,0,0\right) \\ h_2\left(0,1,0,0,0\right) & \bar{h_2}\left(0,-1,0,0,0\right) \\ h_3\left(0,0,1,0,0\right) & \bar{h_3}\left(0,0,-1,0,0\right) \\ h_{45}\left(-\frac{1}{2},-\frac{1}{2},0,0,0\right) & \bar{h_{45}}\left(\frac{1}{2},\frac{1}{2},0,0,0\right) \end{array}$$

- Singlets:

$$\begin{array}{lll} \Phi_{12}\left(-1,1,0,0,0\right) & \bar{\Phi}_{12}\left(1,-1,0,0,0\right) \\ \Phi_{23}\left(0,-1,1,0,0\right) & \bar{\Phi}_{23}\left(0,1,-1,0,0\right) \\ \Phi_{31}\left(1,0,-1,0,0\right) & \bar{\Phi}_{31}\left(-1,0,1,0,0\right) \\ \phi_{45}\left(\frac{1}{2},\frac{1}{2},1,0,0\right) & \bar{\phi}_{45}\left(-\frac{1}{2},-\frac{1}{2},-1,0,0\right) \\ \phi^{+}\left(\frac{1}{2},-\frac{1}{2},0,0,1\right) & \bar{\phi}^{+}\left(-\frac{1}{2},\frac{1}{2},0,0,-1\right) \\ \phi^{-}\left(\frac{1}{2},-\frac{1}{2},0,0,-1\right) & \bar{\phi}^{-}\left(-\frac{1}{2},\frac{1}{2},0,0,1\right) \\ \phi_{3,4}\left(\frac{1}{2},-\frac{1}{2},0,0,0\right) & \bar{\phi}_{3,4}\left(-\frac{1}{2},\frac{1}{2},0,0,0\right) \\ \eta_{1,2}\left(0,0,0,1,0\right) & \bar{\eta}_{1,2}\left(0,0,0,-1,0\right) \end{array}$$

- Hidden Sector:

T: **10** of SO(10); D: **6**, \tilde{F} : **4**, $\tilde{\bar{F}}$: $\overline{\bf 4}$ of SU(4).

The \tilde{F}_i , $\tilde{\tilde{F}}_j$ fields carry $\pm 1/2$ electric charges.

$$T_{1}\left(-\frac{1}{2},0,\frac{1}{2},0,0\right) \\ T_{2}\left(-\frac{1}{2},-\frac{1}{2},0,0,-\frac{1}{2}\right) \\ T_{3}\left(-\frac{1}{2},0,\frac{1}{2},0,0\right) \\ D_{1}\left(0,-\frac{1}{2},\frac{1}{2},\frac{1}{2},0\right) & D_{2}\left(0,-\frac{1}{2},\frac{1}{2},-\frac{1}{2},0\right) \\ D_{3}\left(-\frac{1}{2},0,\frac{1}{2},0,0\right) & D_{4}\left(-\frac{1}{2},-\frac{1}{2},0,0,\frac{1}{2}\right) \\ D_{5}\left(0,-\frac{1}{2},\frac{1}{2},0,0\right) & D_{6}\left(0,\frac{1}{2},-\frac{1}{2},0,0\right) \\ D_{7}\left(\frac{1}{2},0,-\frac{1}{2},0,0\right) & D_{7}\left(\frac{1}{2},0,-\frac{1}{2},0,0\right) \\ \widetilde{F}_{3}^{+}\left(\frac{1}{4},-\frac{1}{4},-\frac{1}{4},0,\frac{1}{2}\right) & \widetilde{F}_{4}^{-}\left(-\frac{1}{4},\frac{3}{4},\frac{1}{4},0,0\right) \\ \widetilde{F}_{5}^{+}\left(-\frac{1}{4},\frac{1}{4},-\frac{1}{4},0,\frac{1}{2}\right) & \widetilde{F}_{6}^{+}\left(-\frac{1}{4},\frac{1}{4},\frac{1}{4},-\frac{1}{4},0,-\frac{1}{2}\right) \\ \widetilde{F}_{3}^{-}\left(\frac{1}{4},-\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{2},-\frac{1}{2}\right) & \widetilde{F}_{4}^{-}\left(-\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4},0,-\frac{1}{2}\right) \\ \widetilde{F}_{5}^{+}\left(-\frac{1}{4},-\frac{1}{4},\frac{1}{4},0,-\frac{1}{2}\right) & \widetilde{F}_{6}^{-}\left(-\frac{3}{4},\frac{1}{4},\frac{1}{4},0,-\frac{1}{2}\right) \\ \widetilde{F}_{5}^{+}\left(-\frac{1}{4},-\frac{1}{4},\frac{1}{4},0,-\frac{1}{2}\right) & \widetilde{F}_{6}^{-}\left(-\frac{3}{4},\frac{1}{4},-\frac{1}{4},0,0\right) \\ \end{array}$$

• The cubic superpotential is given by:

$$W_{3} = g\sqrt{2} \left\{ F_{0}F_{1}h_{1} + F_{2}F_{2}h_{2} + F_{4}F_{4}h_{1} + F_{4}\bar{f}_{5}\bar{h}_{45} + F_{3}\bar{f}_{3}\bar{h}_{3} \right.$$

$$\left. + \bar{f}_{2}l_{2}^{c}h_{2} + \bar{f}_{5}l_{5}^{c}h_{2} \right.$$

$$\left. + \frac{1}{\sqrt{2}}F_{4}\bar{F}_{5}\phi_{3} + \frac{1}{2}F_{4}\bar{F}_{4}\Phi_{0} + \bar{F}_{4}\bar{F}_{4}\bar{h}_{1} + \bar{F}_{5}\bar{F}_{5}\bar{h}_{2} \right.$$

$$\left. + (h_{1}\bar{h}_{2}\Phi_{12} + h_{2}\bar{h}_{3}\Phi_{23} + h_{3}\bar{h}_{1}\Phi_{31} + h_{3}\bar{h}_{45}\bar{\phi}_{45} + \text{h.c.} \right)$$

$$\left. + \frac{1}{2}(\phi_{45}\bar{\phi}_{45} + \phi^{+}\bar{\phi}^{+} + \phi^{-}\bar{\phi}^{-} + \phi_{i}\bar{\phi}_{i} + h_{45}\bar{h}_{45})\Phi_{3} \right.$$

$$\left. + (\eta_{1}\bar{\eta}_{2} + \bar{\eta}_{1}\eta_{2})\Phi_{0} + (\phi_{3}\bar{\phi}_{4} + \bar{\phi}_{3}\phi_{4})\Phi_{5} \right.$$

$$\left. + (\Phi_{12}\Phi_{23}\Phi_{31} + \Phi_{12}\phi^{+}\phi^{-} + \Phi_{12}\phi_{i}\phi_{i} + \text{h.c.} \right)$$

$$\left. + T_{1}T_{1}\Phi_{31} + T_{3}T_{3}\Phi_{31} \right.$$

$$\left. + D_{6}D_{6}\Phi_{23} + D_{1}D_{2}\bar{\Phi}_{23} + D_{5}D_{5}\bar{\Phi}_{23} + D_{7}D_{7}\bar{\Phi}_{31} \right.$$

$$\left. + D_{3}D_{3}\Phi_{31} + \frac{1}{2}D_{5}D_{6}\Phi_{0} + \frac{1}{\sqrt{2}}D_{5}D_{7}\bar{\phi}_{3} \right.$$

$$\left. + \tilde{F}_{4}\tilde{F}_{6}\bar{\Phi}_{12} + \frac{1}{2}F_{3}\tilde{F}_{4}\Phi_{0} + \frac{1}{2}F_{2}\tilde{F}_{5}\Phi_{3} + \tilde{F}_{6}\tilde{F}_{4}\phi^{+} \right.$$

$$\left. + \frac{1}{\sqrt{2}}\tilde{F}_{5}\tilde{F}_{4}\phi_{4} + \tilde{F}_{1}\tilde{F}_{2}D_{5} + \tilde{F}_{2}\tilde{F}_{4}l_{2}^{c} \right\}$$

• The quartic superpotential is given by:

$$W_4 = F_2 \bar{f}_2 \bar{h}_{45} \bar{\phi}_4 + F_3 \bar{F}_4 D_4 D_6 + F_3 \bar{F}_5 D_4 D_7$$

$$+ l_{3}^{c}\tilde{F}_{3}\tilde{\bar{F}}_{6}D_{7} + l_{5}^{c}\tilde{F}_{2}\tilde{\bar{F}}_{3}\bar{\phi}_{3} + \tilde{F}_{1}\tilde{\bar{F}}_{3}(\phi^{+}\bar{\phi}_{3} + \bar{\phi}^{-}\phi_{3})$$

$$+ \tilde{\bar{F}}_{3}\tilde{\bar{F}}_{5}D_{7}\bar{\phi}^{-} + \tilde{F}_{2}\tilde{F}_{5}D_{3}\phi^{-} + \tilde{F}_{2}\tilde{F}_{6}D_{3}\phi_{4} + \tilde{F}_{5}\tilde{\bar{F}}_{1}D_{2}D_{7}$$

$$+ \tilde{F}_{5}\tilde{\bar{F}}_{2}D_{1}D_{7} + \tilde{F}_{3}\tilde{\bar{F}}_{3}D_{3}D_{6} + \tilde{F}_{4}\tilde{\bar{F}}_{3}D_{4}D_{7} + \tilde{F}_{5}\tilde{\bar{F}}_{4}D_{5}D_{7}.$$

Calculable coefficients ($\lambda = c_4 g/M$, $c \sim 1$, $M = 10^{18} \,\text{GeV}$) have been omitted from W_4 but can be calculated using the methods of Ref. [6].

• The Higgs doublet mass matrix is given by:

$$\overline{H}_1$$
 \overline{H}_2 \overline{H}_3 \overline{H}_{45}
 H_1 $\begin{pmatrix} 0 & \Phi_{12} & ar{\Phi}_{31} & 0 \\ ar{\Phi}_{12} & 0 & \Phi_{23} & 0 \\ \Phi_{31} & ar{\Phi}_{23} & 0 & ar{\phi}_{45} \\ 0 & 0 & \phi_{45} & \Phi_{3} \\ L_2 & 0 & 0 & V_2 ar{\phi}_4 \\ L_3 & 0 & 0 & V_3 & 0 \\ L_5 & 0 & 0 & 0 & V_4 \end{pmatrix}$

Note that "Higgs" (H) and "lepton" (L) doublets cannot be distinguished in principle. This result in consistent with Ref. [22] where it is shown that in $SU(5) \times U(1)$ R-parity is not automatically conserved. However, phenomenological constraints require that the bottom portion of the matrix decouples from the top portion [21, 23] and therefore the standard R-parity symmetry is present in the model.

• The Higgs triplet mass matrix is given by:

In this case it is automatic that three d^c states remain light, but which linear combinations these are is model dependent. In fact, mixings among the "canonical" $d_{0,1,2,3,4}^c$ could be an important source of Kobayashi-Maskawa mixing at low energies [24].

- String scenario for a "heavy" top quark:
 - We identify $g\sqrt{2}F_4\bar{f}_5\bar{h}_{45}$ with the top-quark Yukawa coupling, and get $\lambda_t(M_U) = g\sqrt{2}$.
 - At low energies one gets $m_t = \lambda_t(m_t) \sin \beta(174)$ GeV, where $\lambda_t(m_t)$ is the top-quark Yukawa coupling at low energies and $\tan \beta$ is the ratio of the Higgs vacuum expectation values.
 - In Fig. 4 we show the top-quark Yukawa coupling at the unification scale versus m_t (figure from Ref.[25]) for fixed values of $\tan \beta$. The horizontal lines indicate a possible range of string predictions for the top-quark Yukawa coupling.
 - One can see that the experimentally preferred values of m_t ($\sim 170 \pm 10 \,\text{GeV}$) fit well with typical string predictions.

8 Supersymmetry breaking

Since the superpartners of the ordinary particles have not been observed, supersymmetry must be a broken symmetry at low energies. The mechanism of supersymmetry breaking remains unclear, although great strides in this direction have been made in the last few years, especially with inspiration from string theory. The most popular mechanism for dynamical supersymmetry breaking can be outlined as follows:

- The hidden sector gauge group may become strongly interacting at some intermediate scale depending on the hidden gauge group and the hidden matter content. Gaugino condensation will likely then occur, i.e., $\langle \lambda \lambda \rangle \neq 0$.
- The scalar potential must have a minimum which breaks supersymmetry. This appears to require a tuning of two different hidden sector gauge groups with similar gauge and matter content [26]. However, this may not be necessary if the strong interactions are handled in a less than naive way [27].
- The breaking of supersymmetry may be due to the F-term of several possible fields: the dilaton (S), the moduli (T), or the hidden matter (H) fields.
- Since $\langle S \rangle \propto 1/g^2$, the scalar potential must have a minimum in the S direction for a finite value of $\langle S \rangle$. A typical problem is $\langle S \rangle \to \infty$, *i.e.*, $g \to 0$.
- The vacuum energy (value of the scalar potential at the minimum) is the cosmological constant, and therefore should be "small". There is no general solution to this condition, although particular cases may just work out.
- The magnitude of supersymmetry breaking in the observable sector should not exceed $\lesssim 1 \text{ TeV}$. This can be accomplished with suitably chosen hidden sectors [28].

The above scenario for supersymmetry breaking is "string-inspired" but not necessarily consistent with string theory. For example, possible string non-perturbative effects are ignored, and the solution to cosmological constant problem is assumed not to impact the results.

I should also mention another scenario for supersymmetry breaking in string theory, through the so-called Scherk-Schwarz mechanism. In this case supersymmetry is broken perturbatively and its magnitude can come out to be small enough if there is a modulus field which acquires a very large expectation value. This is equivalent to an effective decompactification of a compactified dimension and can be realistic if some conditions are satisfied [29].

Recently a more model-independent approach to string-inspired supersymmetry breaking has become popular [30, 31]. In this approach supersymmetry breaking is parametrized by an angle $\tan \theta = \langle F_S \rangle / \langle F_T \rangle$. Generally one finds that the scalar masses are not universal

$$m_i^2 = m_{3/2}^2 (1 + n_i \cos^2 \theta),$$

where $m_{3/2}$ is the gravitino mass, and n_i is the modular weight of the string state. Non-universality of the first and second-generation scalar masses could easily violate stringent limits on flavor changing neutral currents in the K-system [32]. If one demands universality of the scalar masses, two scenarios arise:

(i)
$$\cos \theta = 0 \Leftrightarrow \langle F_S \rangle \gg \langle F_T \rangle$$
: "dilaton scenario" [30, 31]

$$m_i = m_{3/2}, \quad M_a = \sqrt{3}m_{3/2}, \quad A = -\sqrt{3}m_{3/2}$$

(ii) All n_i equal $(n_i = -1)$. This occurs in $Z_2 \times Z_2$ orbifold models, free-fermionic models, 6 and in the large-T limit of Calabi-Yau compactification. If $\cos \theta = 1 \Leftrightarrow \langle F_T \rangle \gg \langle F_S \rangle$, then $m_i = 0$ and we call this the "**moduli scenario**". More generally: $m_i = \sin \theta \, m_{3/2}, \, M_a = \sqrt{3} \sin \theta \, m_{3/2}$, and $A = -\sqrt{3} \sin \theta \, m_{3/2}$. (If $\sin \theta \to 0$, one needs to worry about one-loop corrections to Kähler potential and gauge kinetic function [31].)

9 The bottom-up approach

- The previous discussion has been mostly about true string phenomenology. However, the subject of supersymmetry breaking already steps into the "string-inspired" phenomenology area, although not completely.
- We now depart from the rigorous string predictions and turn to *string-inspired* phenomenology. In the present context, this consists of taking the best known properties of string models and building a supergravity model based on them. Eventually a single string model may be found where all the desired properties may happen simultaneously. This model would be a true candidate for a fundamental "Theory of Everything".

⁶Detailed studies of this question in realistic free-fermionic models are in progress [33].

- Our string-inspired model consists of:
 - An $SU(5) \times U(1)$ supergravity model whose gauge couplings unify at the string scale $M_{\text{string}} \sim 10^{18} \,\text{GeV}$. This requires one vector-like quark doublet $(M_Q \sim 10^{12} \,\text{GeV})$ and one vector-like quark singlet $(M_D \sim 10^6 \,\text{GeV})$, in addition to the particles in the Minimal Supersymmetric Standard Model. These particles occur in the "search" string $SU(5) \times U(1)$ model of Ref. [21].
 - We also assume that supersymmetry breaking is triggered by a set of soft-supersymmetry-breaking terms which correspond to the "moduli" and "dilaton" universal soft supersymmetry breaking scenarios discussed above.

9.1 Unification of gauge couplings

As just mentioned, the unification of the gauge couplings at the string scale require the existence of new intermediate-mass particles. Their masses depend on the value of the strong coupling, as shown in the following table

$\alpha_3(M_Z)$	$M_D ({ m GeV})$	$M_Q (\mathrm{GeV})$	$\alpha(M_U)$
0.110	$4.9 \times 10^4 \mathrm{GeV}$	$2.2 \times 10^{12} \mathrm{GeV}$	0.0565
0.118	$4.5 \times 10^6 \mathrm{GeV}$	$4.1 \times 10^{12} \mathrm{GeV}$	0.0555
0.126	$2.3 \times 10^8 \mathrm{GeV}$	$7.3 \times 10^{12} \mathrm{GeV}$	0.0547

In figure 5 we show the running of the gauge couplings (solid lines) and their unification at the string scale $M_{\rm string} \sim 10^{18}\,{\rm GeV}$. For reference we also show the case of no intermediate-scale particles (dotted lines) where the gauge couplings unify at a lower scale.

9.2 Soft supersymmetry breaking

We now list all of the soft-supersymmetry-breaking parameters generally allowed in supergravity models. The assumption of *universality* of the soft parameters is also commonly made. This assumption has some basis in the original supergravity models, but is not guaranteed, and in fact it is explicitly violated in most string-inspired supersymmetry breaking models. We also keep a running list of how many parameters are being introduced at each stage.

- Gaugino masses (parameters = 3)
 - $-M_3, M_2, M_1$ parametrize the masses of the superpartners of the gauge bosons of $SU(3)_C, SU(2)_L, U(1)_Y$.
 - The universal soft-supersymmetry-breaking assumption entails: $M_3 = M_2 = M_1 = m_{1/2}$ at M_U . However, there is no phenomenological reason for such requirement, as far as the gaugino masses are concerned.
- Scalar masses (parameters = $5 \times 3 + 2 = 17$)

- Need to provide masses for the squarks and sleptons $(\tilde{Q}, \tilde{U}^c, \tilde{D}^c, \tilde{L}, \tilde{E}^c)_i, i = 1, 2, 3,$ and for the Higgs-boson doublets H_1, H_2 .
- Universality implies that all these masses are equal to m_0 at M_U . Limits on flavor-changing-neutral-currents (FCNCs) at low energies require that the squarks (and sleptons) of the first two generations be nearly degenerate in mass [32]. The universality assumption together with the renormalization group evolution of the scalar masses assures that the experimental limits on FCNCs are easily satisfied. We note that there are no comparable experimental constraints on the third generation sparticles or on the Higgs-boson doublets; nonetheless universality is usually assumed for these masses too.
- Scalar couplings (parameters = 3 + 1 = 4)
 - Each superpotential coupling is accompanied by a soft-supersymmetrybreaking term proportional to it. These soft terms are trilinear couplings among the scalar components of the superfields which appear in the corresponding superpotential terms, as follows

$$\begin{array}{ll} \lambda_t Q t^c H_2 & \to \lambda_t A_t \tilde{Q} \tilde{t}^c H_2 \\ \lambda_b Q b^c H_1 & \to \lambda_b A_b \tilde{Q} \tilde{b}^c H_1 \\ \lambda_\tau L \tau^c H_1 & \to \lambda_\tau A_\tau \tilde{L} \tilde{\tau}^c H_1 \\ \mu H_1 H_2 & \to \mu B H_1 H_2 \end{array}$$

- In this case the universality assumption entails: $A_t = A_b = A_\tau = A$ at M_U .
- All of the soft-supersymmetry-breaking parameters, and the gauge and Yukawa couplings evolve down to low energies as prescribed by the appropriate set of coupled renormalization group equations (RGEs).

9.3 Parameter count at low energies

The following is a list of all the parameters introduced in supersymmetric models (excluding CP-violating phases). The "MSSM" column counts the number of parameters in the Minimal Supersymmetric Standard Model, wherea the "SUGRA" column

refers to the case of supergravity models with universal soft supersymmetry breaking.

Parameter	MSSM	SUGRA	
M_1, M_2, M_3	3	1	$(m_{1/2})$
$(\tilde{Q}, \tilde{U}^c, \tilde{D}^c, \tilde{L}, \tilde{E}^c)_i$	15	1	(m_0)
H_1, H_2	2	0	(m_0)
A_t, A_b, A_{τ}	3	1	(A)
B	1	1	(determined by
μ	1	1	radiative EWx)
$\lambda_{b,t, au}, an eta$	2	2	
	_	_	
Total:	27	7	

- The two minimization conditions of the electroweak scalar potential impose two additional constraints which can be used to determine μ , B and thus reduce the parameter count down to 5 (versus 25 in the MSSM).
- In the two string-inspired scenarios we consider, m_0 and A are known functions of $m_{1/2}$, therefore the parameters are only 3.
- Moreover, in a self-consistent supersymmetry breaking theory, even $m_{1/2}$ (or the relevant scale) would be determined. With the knowledge of m_t , only $\tan \beta$ would remain unknown.

9.4 Determination of the theoretically allowed parameter space

The theoretically allowed parameter space in the variables $(m_t, \tan \beta, m_{1/2}, m_0, A)$ can be determined by a self-consistent procedure of running the RGEs for the various parameters between the weak scale and unification scale and imposing the electroweak breaking constraint. This procedure is non-trivial and is not new [34, 35]. However, because of the revival of supersymmetric grand unification, this procedure has been re-examined in detail prior to the LEP era [36], during the early LEP years [37, 38], and also very recently [39, 40]. Here we just present a "flow chart" of the various steps which are generally followed. This is given in Fig. 6.

The various inputs in the calculation are:

- known quantities: $m_b, m_\tau, \alpha_3, M_Z$
- the top-quark mass m_t
- $\tan \beta$
- the three universal soft-supersymmetry-breaking parameters $m_{1/2}$, m_0 , and A

Note that the parameters are input at different scales.

10 Radiative Electroweak Breaking

10.1 Tree-level minimization

The tree-level Higgs potential, assuming that only the neutral components get vevs, is given by

$$V_0 = (m_{H_1}^2 + \mu^2)h_1^2 + (m_{H_2}^2 + \mu^2)h_2^2 + 2B\mu h_1 h_2 + \frac{1}{8}(g_2^2 + g'^2)(h_2^2 - h_1^2)^2,$$

where $h_i = \text{Re } H_{1,2}^0$. One can then write down the minimization conditions $\partial V_0/\partial h_i = 0$ and obtain

$$\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2,$$

$$B\mu = -\frac{1}{2} \sin 2\beta (m_{H_1}^2 + m_{H_2}^2 + 2\mu^2).$$

The solutions to these equations are physically sensible only if they reflect a minimum away from the origin

$$S = (m_{H_1}^2 + \mu^2)(m_{H_2}^2 + \mu^2) - B^2\mu^2 < 0$$

of a potential bounded from below

$$\mathcal{B} = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 + 2B\mu > 0.$$

Taking the second derivative $\partial^2 V_0/\partial h_i \partial h_j$ one can determine the physical tree-level Higgs masses.

10.2 One-loop minimization

The above procedure is however not completely satisfactory since the tree-level scalar potential has minima which are not renormalization-scale independent. Indeed, in Fig. 7 we show the typical change in shape of the scalar potential as the renormalization group scale is lowered. This variation implies that the vacuum expectation values which give the Z-boson mass vary a lot for scales $Q \lesssim 1 \,\text{TeV}$, as shown schematically in Fig. 8.

The problem is that $\frac{dV_0}{d \ln Q} \neq 0$, that is, the tree-level scalar potential does not satisfy the renormalization group equation, and Q-dependence is present. The solution to this problem is to use instead the one-loop effective potential

$$V_1 = V_0 + \Delta V$$

with

$$\Delta V = \frac{1}{64\pi^2} \operatorname{Str} \mathcal{M}^4 \left(\ln \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right),$$

where $\operatorname{Str} \mathcal{M}^2 = \sum_j (-1)^{2j} (2j+1) \operatorname{Tr} \mathcal{M}_j^2$. This potential satisfies $\frac{dV_1}{d \ln Q} = 0$ (to one-loop order) and the Q-dependence can be minimized [38].

The vevs are now Q-independent (up to two-loop effects) in the range of interest ($\lesssim 1 \text{ TeV}$). However, one must perform the minimization of the potential numerically (non-trivial) and all the spectrum enters into ΔV (although \tilde{t}, \tilde{b} are the dominant contributions). This method also gives automatically the one-loop corrected Higgs boson masses (taking second derivatives of V_1).

10.3 Radiative Symmetry Breaking

Let us now examine how the symmetry is actually broken in the simple (and unrealistic) case of $\mu = 0$ and just considering the tree-level potential. In this case the S < 0 condition (minimum away from the origin) reduces to

$$S \to m_{H_1}^2 \cdot m_{H_2}^2 < 0,$$

and we must arrange that one $m_H^2 < 0$ somehow.

Consider the relevant RGEs schematically (setting $\lambda_b = \lambda_\tau = 0$)

$$\frac{d\widetilde{m}^2}{dt} = \frac{1}{(4\pi)^2} \left\{ -\sum_i c_i g_i^2 M_i^2 + c_t \lambda_t^2 \left(\sum_i \widetilde{m}_i^2 \right) \right\}$$

with coefficients

The runnings of the scalar masses are shown in Fig. 9 for a particular choice of the parameters, although the qualitative result is correct in most cases of interest. We observe:

- $m_{H_1}^2 < 0$, and $m_{H_2}^2 < 0$ for $Q < Q_0$
- $m_{\tilde{Q},\tilde{U}^c,\tilde{D}^c}^2 > 0$ because of the large α_3 contribution in their RGEs, and the smaller λ_t dependence.

Therefore, the electroweak symmetry is broken "radiatively" and the squark and slepton squared masses remain positive.

11 The allowed parameter space

We consider the two string-inpired soft-supersymmetry-breaking scenarios discussed above:

- "moduli" scenario $m_0 = A = 0$ [23]
- "dilaton" scenario $m_0 = \frac{1}{\sqrt{3}} m_{1/2}$, $A = -m_{1/2}$ [41] (This scenario has also been considered in the context of the MSSM in Ref. [42].)

There are only three parameters:

- $m_{1/2} \leftrightarrow m_{\tilde{g}} \leftrightarrow m_{\chi_1^{\pm}}$
- $\tan \beta$

•
$$m_t^{\text{pole}} = \begin{cases} > 131 \,\text{GeV} & \text{D0 [43]} \\ 160 \pm 13 \,\text{GeV} & \text{EW fits for light Higgs [44]} \\ 174 \pm 17 \,\text{GeV} & \text{CDF [45]} \end{cases}$$

Parameter space:

- We fix $m_t = 150, 170 \,\text{GeV}$, and vary $\tan \beta$ and $m_{\chi_1^{\pm}}$.
- Note: the "pole" mass (m_t^{pole}) , as measured experimentally) is 5% higher than the "running" mass (m_t) which we use here. Therefore, our m_t choices correspond to $m_t^{\text{pole}} \approx 157,178\,\text{GeV}$.
- We keep only points which satisfy all LEPI bounds on sparticle and Higgs-boson masses $(m_{\chi_1^{\pm}} > 45 \,\text{GeV}, \, \Gamma_Z^{inv}, \, m_h \gtrsim 60 \,\text{GeV}, \, m_{\tilde{l}} \gtrsim 45 \,\text{GeV})$ [38].
- The lightest supersymmetric particle (LSP) is the lightest neutralino χ , with a calculated relic abundance $\Omega_{\chi}h_0^2 < 1$. Thus, cosmological constraints are automatically satisfied.

The parameter spaces are shown in Fig. 10 and Fig. 11 for the moduli and dilaton scenarios respectively [46]. The allowed points in parameter space are marked by various symbols. Excluded points are blank. Further experimental constraints apply, as discussed below, and lead to further excluded points (all points with symbols other than a period).

12 Experimental constraints

We now discuss further experimental constraints which restric the parameter spaces in the moduli and dilaton scenarios.

12.1 $b \rightarrow s \gamma$

- There are three main contributions to this process (see Fig. 12): (i) the W-t loop, (ii) the $H^{\pm}-t$ loop, and the (iii) the $\chi^{\pm}-\tilde{t}$ loop. The first two contributions are negative, whereas the last one could have either sign. In fact, its sign is strongly correlated with the sign of μ .
- In $SU(5)\times U(1)$ supergravity these contributions have been calculated in Ref. [47] and are shown in Fig. 13 for the moduli scenario. For $\mu > 0$ one can observe the destructive interference effects. The horizontal lines correspond to the latest CLEOII limits $B(b \to s\gamma) = (0.6 5.4) \times 10^{-4}$ at 95%CL [48].
- One should be aware that one-loop QCD corrections change the tree-level result by a large factor, thus two-loop QCD corrections are expected to be large as well. The lack of a complete two-loop calculation is the largest source of uncertainty in this calculation.
- However, since supersymmetric contributions can be much larger or much smaller than the Standard Model prediction (which depends only on m_t , see $\mu < 0$ in Fig. 13), a measurement of $B(b \to s\gamma)$ will (and already has) constrain(ed) the parameter space in important ways. The excluded points of parameter space are denoted by pluses (+) in Figs. 10,11.

12.2 $(g-2)_{\mu}$

- The supersymmetric one-loop contributions to the anomalous magnetic moment of the muon $(g-2)_{\mu}$ are shown in Fig. 14, and have been calculated in Ref. [49]. The results in the moduli scenario are shown in Fig. 15.
- The present experimental value for $(g-2)_{\mu}$ is $a_{\mu}^{exp}=1165923\,(8.5)\times 10^{-9}$ [50], whereas the latest Standard Model prediction is $a_{\mu}^{SM}=1165919.20\,(1.76)\times 10^{-9}$ [51]. From these two numbers we get an allowed interval (at 95%CL) for any beyond-the-standard-model contribution:

$$-13.2 \times 10^{-9} < a_{\mu}^{susy} < 20.8 \times 10^{-9}.$$

- The figure indicates that this limit is easily violated for not so small values of $\tan \beta$. In fact, there is a significant enhancement in $(g-2)_{\mu}$ for large $\tan \beta$ [49]. Points presently excluded are denoted by crosses (×) in Figs. 10,11.
- The new Brookhaven E821 experiment [52] expects to reach an ultimate sensitivity of 0.4×10^{-9} , and is slated to start taking data in January of 1996.
- This new sensitivity is designed to test the electroweak contribution to $(g-2)_{\mu}$, which is much smaller than the typical supersymmetric contribution. Therefore we expect very important restrictions on the parameter space of (or indirect evidence for) supersymmetric models.

12.3 ϵ_1, ϵ_b

- The one-loop electroweak corrections to the LEP observables can be parametrized in terms of four quantities: $\epsilon_{1,2,3,b}$ [53]. Of these, only ϵ_1 (related to the ρ -parameter) and ϵ_b (related to $Z \to b\bar{b}$) have been constraining at all over the running of LEP. Both these parameters have a quadratic dependence on m_t ; ϵ_1 also has a logarithmic dependence on the Higgs-boson mass.
- The main diagrams contributing to these parameters in supersymmetric models are shown in Figs. 16 and 17. In both cases, the effects of supersymmetry are most significant for a light chargino $m_{\chi_1^{\pm}} \lesssim 60-70\,\text{GeV}$, which shifts ϵ_1 negatively and thus compensates for a large top-quark mass. An example of this effect on ϵ_1 is shown in Fig. 18. Otherwise supersymmetry decouples completely and the Standard Model results with a light Higgs boson are obtained.
- In Fig. 19 [46] we show the calculated values of ϵ_1 and ϵ_b in the $SU(5) \times U(1)$ model with moduli scenario. The various experimental ellipses indicate a preference for lighter top-quark masses. The size of the ellipses is expected to be reduced by a factor of two with the 93+94 LEP data.

12.4 Neutrino telescopes

- Neutralinos (χ) in the galactic halo are captured by the Sun and the Earth and eventually annihilate $\chi\chi \to f\bar{f} \to \cdots \to \text{into high-energy }\nu$'s. These neutrinos can then travel to underground (or underwater) detectors, such as Kamiokande, MACRO, Amanda, Nestor, Dumand.
- The signal is that of upwardly-moving muon fluxes in the detector which are above the expected atmospheric neutrino background. At present there are only flux limits from Kamiokande, although limits from MACRO are forthcoming.
- The concentration of Fe⁵⁶ nuclei on Earth enhances the capture of neutralinos with mass close to 56 GeV. In Fig. 20 [54] we show the predicted flux for Earth capture and the present Kamiokande upper limit. One can see the Fe⁵⁶ enhancement and that the data already impose some (although small) restrictions on parameter space. The excluded points are denoted by diamonds (\$\diamonds\$) in Figs. 10,11.
- It is expected that once MACRO is fully operational, an improvement in flux sensitivity by a factor of 2–10 would be achieved.

13 Prospects for direct detection

We now discuss the prospects for direct detection of the sparticles and Higgs bosons in the string-inspired $SU(5) \times U(1)$ models which we have discussed above. The parameter space which is explored is that which is allowed by all of the experimental constraints introduced in the previous section.

13.1 Tevatron

- The missing energy signature in \tilde{q} , \tilde{g} production is kinematically disfavored since one generally obtains $m_{\tilde{q}} \approx m_{\tilde{g}} \gtrsim 250 \,\text{GeV}$ in $SU(5) \times U(1)$ supergravity.
- A lot more accessible is the *trilepton* channel [56]: $p\bar{p} \to \chi_2^0 \chi_1^{\pm} X \to 3l$. In these models one typically obtains large leptonic branching fractions for the charginos $B(\chi_1^{\pm} \to e + \mu) \approx 2/3$, whereas $B(\chi_2^0 \to ee, \mu\mu)$ can be small for light charginos.
- In Fig. 21 we show the rate for trilepton events versus the chargino mass in the $SU(5) \times U(1)$ "moduli scenario" (the results are somewhat smaller in the "dilaton scenario") [57, 46]. The solid line is the present CDF upper limit on the trilepton rate [58].
- Experimental efficiencies for detection of trilepton events are small (< 10%) because of large "instrumental" backgrounds, *i.e.*, when jets "fake" leptons in the detector.
- By the end of run IB (1993–95) it is expected that $\sim 100 \,\mathrm{pb}^{-1}$ of data would be collected by each detector. This should move the experimental limit from the solid line in Fig. 21 down to the dashed line, *i.e.*, probing chargino masses as high as 100 GeV. Points in parameter space reachable with this improved sensitivity are shown as pluses (+) in Fig. 22 [46].

13.2 LEPII

We discuss three supersymmetry signatures at LEPII: Higgs boson production, chargino pair production, and selectron pair production.

13.2.1 Lightest Higgs boson

- The dominant production mechanism is: $e^+e^- \to Z^* \to Zh(h \to b\bar{b})$, where the Higgs boson decays into two *b*-jets and *b*-tagging is used to reduce the light-jet background.
- The cross section for the supersymmetric process is proportional to the corresponding Standard Model cross section: $\sigma_{susy} = \sin^2(\alpha - \beta)\sigma_{SM} \approx \sigma_{SM}$, where the last result follows in models with

- radiative breaking, *i.e.*, the lightest supersymmetric Higgs boson looks a lot like the Standard Model Higgs boson.
- The relevant branching fraction $B(h \to b\bar{b}) \approx B(H_{SM} \to b\bar{b})$, except when the $h \to \chi_1^0 \chi_1^0$ channel is open (for $\lesssim 10\%$ of the points) in which case the expected signal can be greatly eroded.
- The cross section for Higgs boson production at LEPII for $\sqrt{s} = 200$ and 210 GeV are shown in Fig. 23 [46]. The accumulation of points corresponds to the Standard Model result; the points "falling off" the curves correspond to the opening of the $h \to \chi_1^0 \chi_1^0$ channel.
- The LEPII sensitivity is expected to be $\gtrsim (0.1-0.2)$ pb (dashed lines in Fig. 23) [59], which implies a mass reach of $m_h \lesssim \sqrt{s} 95$.
- We note that the Higgs-boson mass is the most directly useful piece of information that could come out of LEPII. This is shown in Fig. 24 [46], where the Higgs-boson mass contours are given and show that once m_h is known, $\tan \beta$ would be determined in terms of the chargino mass.

13.2.2 Charginos

- The production channel is: $e^+e^- \to \chi_1^+\chi_1^- \to 1l+2j$, where the 1-lepton+2-jets signature (i.e., the "mixed" signal) is used.
- A problem with this channel occurs when $B(\chi_1^{\pm} \to l) \approx 1$ and the chargino branching fraction into jets is strongly suppressed. This phenomenon occurs in the $SU(5) \times U(1)$ models we consider.
- The expected experimental sensitivity (5σ signal over background) is $(\sigma B)_{mixed} \gtrsim 0.05 \text{ pb}$ with $\mathcal{L} = 500 \text{ pb}^{-1}$ [60].
- In $SU(5) \times U(1)$ supergravity this discovery channel has been first studied in Ref. [61]. The points in the still-allowed parameter space which are reachable through this mode are denoted by crosses (\times) in Fig. 22.

13.2.3 Sleptons

- The production channel is: $e^+e^- \to \tilde{e}\tilde{e} \to e\not e p$, where the lightest (right-handed) selectron is the one dominantly produced.
- The experimental sensitivity (at the 5σ level) is expected to be $(\sigma B)_{dilepton} \gtrsim 0.21 \text{ pb}$ with $\mathcal{L} = 500 \text{ pb}^{-1}$ [62, 46]. The reason for this large background is the irreducible background cross section $\sigma(e^+e^- \to W^+W^- \to 2l) = 0.9 \text{ pb}$.
- The points in the still-allowed parameter space which are reachable through this mode are denoted by crosses (\$\dphi\$) in Fig. 22 [61, 46].

- Note that selectrons consitute a deeper probe of the parameter space than direct chargino searches (in the no-scale scenario). Similar remarks apply to $\tilde{\mu}$ and $\tilde{\tau}$ pair production, although the cross sections are somewhat smaller because of the loss of the t-channel diagrams. (Sleptons are too heavy to be observable at LEPII in the dilaton scenario.)
- Note that if the Tevatron (1994–95) sees charginos, then LEPII (1996) will be in business.

13.3 DiTevatron

- In the wake of the SSC demise, there has been a recent proposal to upgrade the Tevatron [63, 64, 65] as follows:
 - $-p\bar{p}$ collisions at $\sqrt{s} = 4 \,\text{TeV}$
 - $\mathcal{L} = 2 \times 10^{32} \text{cm}^{-2} \text{s}^{-1} \to \int \mathcal{L} \sim 2 \, \text{fb}^{-1}/\text{year}.$
 - Would use single ring of SSC magnets, sized down to fit the Tevatron tunnel. The Tevatron would be used for injection. The detectors would need to be upgraded (as currently planed) but not replaced.
 - If this plan is approved, the new rings of magnets should be installed when the Main Injector is put in place, and the machine could be doing physics before the LHC turns on.
- We have studied the possible reach of this machine for charginos via the trilepton mode, squarks and gluinos via the missing energy signature, and Higgs bosons produced in association with a W or Z boson [66]. With an integrated luminosity of 5 fb⁻¹, the reach for chargino masses is expected to be 210 (150) GeV in the moduli (dilaton) scenario. This is exemplified in Figs. 25,26 where the corresponding reach at the Tevatron is also shown. The corresponding reach in squark and gluino masses is estimated to be ~ 700 GeV and is depicted in Fig. 27 in terms of the significance for such a signal. The lightest Higgs boson could also be searched up to a mass of ~ 120 GeV [67, 66].
- All in all, the doubling of the Tevatron energy to the DiTevatron should allow one to probe a large fraction of parameter space of $SU(5) \times U(1)$ supergravity.

14 Conclusions

- True string phenomenology is very powerful. Once a vacuum is singled out, every parameter of the model is in principle calculable.
- We have discussed a two-parameter (plus m_t), very predictive, string-inspired $SU(5) \times U(1)$ supergravity model.

- With the string-inspired assumptions for the supersymmetry breaking scenarios, several experimental tests have been worked out in detail.
- Experimental outlook

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1994:
   Tevatron (trileptons)
   CLEO (b \to s\gamma)
   LEPI (m_h)

1996:
   Brookhaven (g-2)_{\mu}
   LEPII (Higgs, charginos, sleptons)
   MACRO (neutralinos)

2000(?):
   DiTevatron (charginos, squarks, gluinos, Higgs)

2005(?):
   LHC (Higgs, squarks, gluinos, charginos)
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- We should remark that the "real" string model, which incorporates all the features we would like to have in a supergravity model, is yet to be built.
- Realistic supersymmetry breaking scenarios derived from string remain as the largest stumbling block to a "Theory of Everything".

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Figure Captions

- 1. The external string states are mapped onto the world-sheet by a conformal transformation. The crosses represent the vertex operators which describe the mapped string states.
- 2. String perturbation theory is an expansion in the topology of the two-dimensional world-sheet. Higher orders in perturbation theory are represented by surfaces with increasingly larger number of handles. Note that at each order in perturbation theory there is only one string diagram.
- 3. Dimension five operator mediating proton decay in $SU(5) \times U(1)$. This diagram is suppressed because of the lack of H_3 , \bar{H}_3 mixing.
- 4. The top-quark Yukawa coupling at the unification scale versus m_t for fixed values of $\tan \beta$. The horizontal lines indicate a possible range of string predictions for the top-quark Yukawa coupling.
- 5. The running of the gauge couplings in $SU(5) \times U(1)$ supergravity for $\alpha_3(M_Z) = 0.118$ (solid lines). The intermediate-scale particle masses have been derived using the gauge coupling RGEs to achieve unification at $M_U = 10^{18}$ GeV. The case with no intermediate-scale particles (dotted lines) is also shown; here $M_U \approx 10^{16}$ GeV.
- 6. A "flow chart" of the various steps typically followed in determining the theoretically allowed parameter space in a supergravity model. The input parameters are m_t , $\tan \beta$, $m_{1/2}$, m_0 , A.
- 7. Typical variation of the tree-level scalar potential as the renormalization scale Q is decreased. One starts with no minimum for $Q > Q_0$, and ends up with runaway minima for $Q < Q_1$.
- 8. Schematic variation of the minima of the tree-level Higgs potential as the renormalization scale Q is lowered.
- 9. Running of the scalar squared masses in supergravity for a typical choice of model parameters (indicated). Note that the electroweak symmetry is broken radiatively, and the squark and slepton squared masses remain positive.
- 10. The parameter space for no-scale $SU(5) \times U(1)$ supergravity (moduli scenario) in the $(m_{\chi_1^{\pm}}, \tan \beta)$ plane for $m_t = 150, 170 \,\text{GeV}$. The periods indicate points that passed all theoretical and experimental constraints, the pluses fail the $b \to s\gamma$ constraint, the crosses fail the $(g-2)_{\mu}$ constraint, the diamonds fail the neutrino telescopes (NT) constraint, the squares fail the $\epsilon_1 \epsilon_b$ constraint, and the octagons fail the updated Higgs-boson mass constraint. The reference

- dashed line highlights $m_{\chi_1^{\pm}} = 100\,\mathrm{GeV}$, which is the direct reach of LEPII for chargino masses. Note that when various symbols overlap a more complex symbol is obtained.
- 11. The parameter space for no-scale $SU(5) \times U(1)$ supergravity (dilaton scenario) in the $(m_{\chi_1^{\pm}}, \tan \beta)$ plane for $m_t = 150, 170 \,\text{GeV}$. The periods indicate points that passed all theoretical and experimental constraints, the pluses fail the $b \to s \gamma$ constraint, the crosses fail the $(g-2)_{\mu}$ constraint, the diamonds fail the neutrino telescopes (NT) constraint, the squares fail the $\epsilon_1 \epsilon_b$ constraint, and the octagons fail the updated Higgs-boson mass constraint. The reference dashed line highlights $m_{\chi_1^{\pm}} = 100 \,\text{GeV}$, which is the direct reach of LEPII for chargino masses. Note that when various symbols overlap a more complex symbol is obtained.
- 12. The largest one-loop contributions to the $b \to s\gamma$ process in supersymmetric models. The last diagram can interfere destructively with the first two for $\mu > 0$.
- 13. The calculated values of $B(b \to s\gamma)$ in the moduli scenario. Note the destructive interference effect for $\mu > 0$. The present CLEOII limits are as indicated.
- 14. The one-loop supersymmetric contributions to the anomalous magnetic moment of the muon.
- 15. The calculated values of $(g-2)_{\mu}$ in the moduli scenario. Note that large enhancements occur for not too small values of $\tan \beta$.
- 16. Main diagrams contributing to the parameter ϵ_1 in supersymmetric models.
- 17. Main diagrams contributing to the parameter ϵ_b in supersymmetric models.
- 18. An example of the effect of a light chargino on the electroweak parameter ϵ_1 . A light chargino shifts ϵ_1 negatively and thus compensates for a large top-quark mass.
- 19. The calculated values of ϵ_1 and ϵ_b in the $SU(5) \times U(1)$ model with moduli scenario. The various experimental ellipses indicate a preference for lighter top-quark masses. The size of the ellipses is expected to be reduced by a factor of two with the 93+94 LEP data.
- 20. The predicted upwardly-moving muon flux for Earth capture of galactic halo neutralinos, and the present Kamiokande flux upper limit. One can see the Fe⁵⁶ capture enhancement, and that the data already impose some (although small) restrictions on parameter space. An improvement in sensitivity by a factor of 2–10 is expected with MACRO.

- 21. The rate for trilepton events versus the chargino mass in the $SU(5) \times U(1)$ "moduli scenario" (the results are somewhat smaller in the "dilaton scenario").
- 22. The remaining allowed parameter space in $SU(5) \times U(1)$ supergravity in the moduli (top plots) and dilaton (bottom plots) scenarios. Points accesible by trilepton searches are denoted by pluses (+), whereas those accessible by chargino and selectron searches at LEPII are donoted by crosses (×) and diamonds (\diamond), respectively. The contours are of the lightest Higgs boson mass.
- 23. The cross section for Higgs boson production at LEPII for $\sqrt{s}=200$ and 210 GeV. The accumulation of points corresponds to the Standard Model result; the points "falling off" the curves correspond to the opening of the $h \to \chi_1^0 \chi_1^0$ channel.
- 24. The Higgs-boson mass contours in the $(m_{\chi_1^{\pm}}, \tan \beta)$ plane for both scenarios in $SU(5) \times U(1)$ supergravity. Once m_h is known, $\tan \beta$ would be determined as a function of the chargino mass. The Higgs-boson mass would be the most useful piece of information to come out of LEPII.
- 25. Trilepton yield $(\sigma \times B)$ versus chargino mass in chargino production in $p\bar{p}$ collisions. The dots define the range of parameters allowed within the string-inspired $SU(5)\times U(1)$ supergravity model (for $m_t^{\rm pole}=157\,{\rm GeV}$) with moduli scenario for supersymmetry breaking. Results are shown for each sign of the Higgs mixing parameter μ . The upper (lower) plots show the limits which could be reached at the Tevatron (DiTevatron). The sensitivity limits are for the indicated integrated luminosities .
- 26. Trilepton yield $(\sigma \times B)$ versus chargino mass in chargino production in $p\bar{p}$ collisions. The dots define the range of parameters allowed within the string-inspired $SU(5) \times U(1)$ supergravity model (for $m_t^{\text{pole}} = 157 \,\text{GeV}$) with dilaton scenario for supersymmetry breaking. Results are shown for each sign of the Higgs mixing parameter μ . The upper (lower) plots show the limits which could be reached at the Tevatron (DiTevatron). The sensitivity limits are for the indicated integrated luminosities.
- 27. Statistical significance for gluino and squark events at the DiTevatron with $\mathcal{L} = 5 \,\mathrm{fb}^{-1}$. (The significance varies with \mathcal{L} as $\sqrt{\mathcal{L}}$.) These eventes were selected by the criteria $p_{\perp} > 150 \,\mathrm{GeV}$, and 4 jets with $p_{\perp} > 40 \,\mathrm{GeV}$. Bands are shown for signal S from gluino pairs, and squark/gluino combinations for parameters which are consistent with the minimal SU(5) supergravity model and the $SU(5) \times U(1)$ supergravity models, respectively. The background B is calculated from $Z \to \nu \bar{\nu}$; the bands provide for a factor of 5 deterioration of S/B ratio due to additional backgrounds or inefficiencies.

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